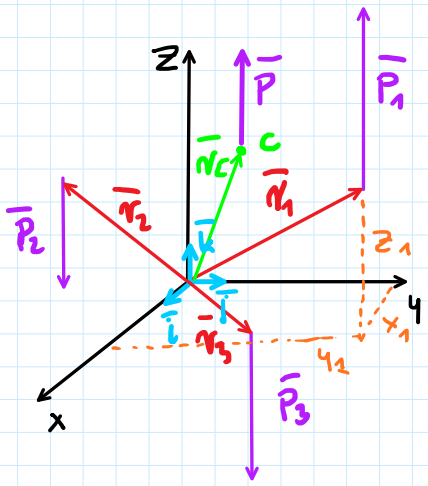
Środki ciężkości



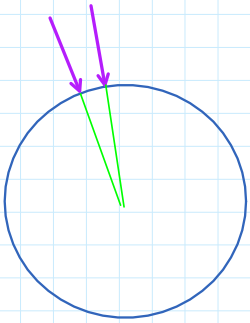
Układ sił równoległych

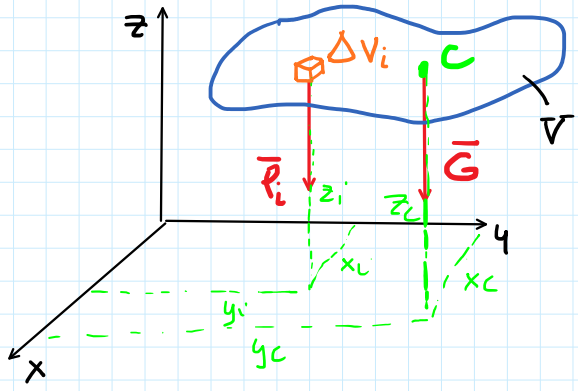
$$\bar{P} = \sum_{i=1}^{n} \bar{P}_i$$

$$\bar{r}_i = \bar{i} \cdot x_i + \bar{j} \cdot y_i + \bar{k} \cdot z_i$$

$$\bar{r}_c \cdot \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \bar{r}_i P_i$$

$$\bar{r}_c = \frac{\sum \bar{r}_i \cdot P_i}{\sum P_i} \Rightarrow \begin{cases} x_c = \dfrac{\sum x_i P_i}{P} \\ y_c = \dfrac{\sum y_i P_i}{P} \\ z_c = \dfrac{\sum z_i P_i}{P} \end{cases}$$





$$P_i = \Delta V_i \cdot \gamma_i \qquad m^3 \cdot \frac{N}{m^3} = N$$

$$x_c = \frac{\sum P_i \cdot x_i}{\sum P_i} = \frac{\sum \Delta V_i \cdot \gamma_i \cdot x_i}{\sum \Delta V_i \cdot \gamma_i}$$

$$\Delta V \to 0 \qquad i \to \infty$$

$$x_c = \frac{\int_V \gamma x \, dV}{\int_V \gamma \, dV}, \quad y_c = \frac{\int_V \gamma y \, dV}{\int_V \gamma \, dV}, \quad z_c = \frac{\int_V \gamma z \, dV}{\int_V \gamma \, dV}$$

$$x_c = \frac{\int_V \gamma x \, dV}{\int_V \gamma \, dV} = \frac{\int_V \gamma x \, dV}{G}$$

$$G = mg, \quad \gamma = g \cdot \rho$$

ρ – ciężar właściwy

g – gęstość

$$x_c = \frac{\int_V g \rho x \, dV}{mg} = \frac{g \int_V \rho x \, dV}{mg} = \frac{\int_V \rho x \, dV}{m}$$

$$\begin{cases} S_{yz} = mx_c = \int_V \rho x \, dV \\ S_{xy} = mz_c = \int_V \rho z \, dV \\ S_{xz} = my_c = \int_V \rho y \, dV \end{cases}$$

Moment statyczny

$$x_c = \frac{\int_V x\gamma\,dV}{\int_V \gamma\,dV} = \frac{\gamma\int_V x\,dV}{\gamma\int_V dV} = \frac{\int_V x\,dV}{V} \Rightarrow$$

$$x_c V = \int_V x\,dV = S_{yz}$$

$$y_c V = \int_V y\,dV = S_{zx}$$

$$z_c V = \int_V z\,dV = S_{xy}$$

$$x_c = \frac{S_{zy}}{V},\quad y_c = \frac{S_{zx}}{V},\quad z_c = \frac{S_{xy}}{V}$$ dla 3D

$$x_c = \frac{S_y}{A},\quad y_c = \frac{S_x}{A}$$ dla 2D

METODY DLA BRYŁ CIĘŻKOŚCI:

* ANALITYCZNIE
* METODA MOMENTÓW STATYCZNYCH
* METODA PODZIAŁU
  - ELEMENTY PROSTE → SKOMPLIKOWANYCH
* METODA SYMETRII
* METODA MAS UJEMNYCH